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## Non-existence of dark solitons in a nonlinear Schrödinger–Maxwell–Bloch fibre system

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**Abstract.** We consider the coupled system of nonlinear Schrödinger and Maxwell–Bloch (NLS–MB) equations, which govern the nonlinear pulse propagation in erbium doped optical fibres. With the help of the Painlevé singularity structure analysis, we prove the non-existence of optical solitons in the NLS–MB fibre system in the normal dispersion regime.

### 1. Introduction

Currently many theoretical works concentrate mainly on the practical feasibility of optical solitons. One such important system is the coupled version of nonlinear Schrödinger and Maxwell–Bloch (NLS–MB) equations. Recently many researchers have worked on this, achieving many results [1–6].

In an optical system, two types of soliton are possible. One is governed by the NLS equation, which is a balance between the group velocity dispersion (GVD) and the self-phase modulation (SPM) due to the Kerr nonlinearity in optical fibres. The resulting dynamical equation for the NLS solitons is of the form [7–10]

$$q_z = i\beta q_{tt} - i\gamma |q|^2 q \quad (1)$$

where  $q$  represents the complex envelope amplitude,  $t$  and  $z$  are the time and distance along the direction of propagation,  $\beta$  is the second derivative of the axial wavenumber  $k$  with respect to the angular frequency  $\omega_0$  and describes the GVD and  $\gamma = n_2\omega_0/cA_{\text{eff}}$  is the SPM parameter, where  $n_2$  is the Kerr coefficient,  $c$  is the speed of light and  $A_{\text{eff}}$  is the effective core area of the fibre.

The GVD parameter  $\beta$  can take both positive and negative values with respect to the central frequency of the carrier wave. Hence for positive and negative values of  $\beta$  the fibre system will be working either in the normal dispersion regime or in the anomalous dispersion regime respectively. NLS solitons in normal and anomalous dispersion regimes are called dark [8] and bright solitons [7] respectively.

Dark solitons [11] are generally considered to be less desirable for applications in high-speed communication systems because of their higher average power and resulting undesirable effects, such as excitation of stimulated Brillouin back-scattering. On the other hand, bright solitons have a drawback of fully utilizing the line capacity because of the necessity of keeping relatively large separations between pulses to avoid accumulation of bit errors. Also, optical

losses decrease the intensity of the pulse, along with a corresponding increase in the width. This effect is smaller in the dark-optical-soliton case. It was shown both numerically and analytically that the time jitter is lower in a dark soliton than in the corresponding bright soliton [12, 13]. The interactive force between two dark solitons is always repulsive, unlike the bright-soliton case, and decreases twice as fast as a function of the distance between the solitons. The separation increases monotonically rather than periodically as in the case of bright solitons.

The other type of optical soliton is the MB soliton in two-level resonant media. In 1967, McCall and Hahn [14] explained a special type of lossless pulse propagation in two-level resonant media (for instance erbium atoms). They showed that if the energy difference between the two levels of the medium coincides with the optical wavelength, then coherent absorption takes place. The medium becomes optically transparent to that particular wavelength, called the self-induced transparency (SIT). MB equations explain the process of SIT. MB equations take the form [14, 15]

$$\begin{aligned} q_z &= \langle p \rangle \\ p_t &= i\omega p - f q \eta \\ \eta_t &= 2f(qp^* + q^*p) \end{aligned} \quad (2)$$

where  $p$  and  $\eta$  are given by  $v_1 v_2^*$  and  $|v_1|^2 - |v_2|^2$  respectively ( $v_1$  and  $v_2$  are wavefunctions of two energy levels of erbium atoms) and  $f$  describes the character of interaction between the propagating field and the two-level resonant atoms. The bracketed term  $\langle \dots \rangle$  means, averaging over the entire frequency range,

$$\begin{aligned} \langle p(z, t; \omega) \rangle &= \int_{-\infty}^{\infty} p(z, t; \omega) g(\omega) d\omega \\ \int_{-\infty}^{\infty} g(\omega) d\omega &= 1 \end{aligned} \quad (3)$$

where  $g(\omega)$  is the uncertainty in the energy levels of the erbium atoms.

Very recently, Opatrný *et al* [16] have reported interesting results on dark and bright solitons in resonantly absorbing gratings, with a different form of the MB equations. Physical context considered in this article is totally different from the one considered in [16].

If the fibres are doped with erbium atoms, then SIT can also be induced in optical fibres. This type of soliton pulse propagation was shown for the first time by Maimistov and Manykin [1] in 1983. Nakazawa *et al* [17, 18] experimentally observed the coexistence of NLS solitons and MB solitons in erbium doped resonant fibres. In [19–21], the possibility of coexistence of the NLS soliton and the MB soliton with some higher-order terms are also shown. The NLS–MB equations are given by

$$\begin{aligned} q_z &= i\beta q_{tt} - i\gamma |q|^2 q + \langle p \rangle \\ p_t &= i\omega p - f q \eta \\ \eta_t &= 2f(qp^* + q^*p). \end{aligned} \quad (4)$$

Here, in this article, we prove the non-existence of the dark optical solitons in the coupled system of the NLS equation and the MB equations using the Painlevé singularity structure analysis and, finally, we discuss the impossibility of propagating dark optical solitons in an erbium doped fibre system governed by the NLS–MB equations.

## 2. Painlevé analysis

The Painlevé analysis is a powerful method in nonlinear science for establishing the integrability of a given nonlinear partial differential equation, that is, solutions which are

free from movable critical manifolds [22]. This analysis is also useful to derive the parametric conditions for the integrability of the system equation. Here, we carry out Painlevé analysis of the NLS–MB equations (4) to derive the condition between the fibre parameters  $\beta$ ,  $\gamma$  and  $f$ . Here it is important to mention that the signum of  $\beta$  only determines the type of dispersion in which the pulse is propagating in optical fibres, so the parametric condition on the signum of  $\beta$  for the integrability of system equation (4) will help in finding the possibilities for the existence of dark and bright solitons in NLS–MB fibre systems.

Because of the averaging term  $\langle p \rangle$ , as such, equation (4) cannot be studied from the Painlevé analysis point of view. Thus for mathematical convenience, the line-shape function  $g(\omega)$  in equation (3) is considered to be a Dirac delta function at resonant frequency  $\omega_0$ , so the averaging function reduces to

$$\begin{aligned} \langle p(z, t; \omega) \rangle &= \int_{-\infty}^{\infty} p(z, t; \omega) \delta(\omega - \omega_0) d\omega \\ &= p(z, t; \omega_0). \end{aligned} \tag{5}$$

A new set of variables,  $a (=q)$ ,  $b (=q^*)$ ,  $c (=p)$ ,  $d (=p^*)$  and  $e (=η)$ , is introduced for the purpose of Painlevé singularity structure analysis. Thus, using equations (4) and (5),  $a$ ,  $b$ ,  $c$ ,  $d$  and  $e$  can be written as

$$\begin{aligned} a_z &= i\beta a_{tt} - i\gamma a^2 b + c \\ b_z &= -i\beta q_{tt} + i\gamma b^2 a + d \\ c_t &= i\omega_0 c - fae \\ d_t &= -i\omega_0 d - fbd \\ e_t &= 2f(ac + bd). \end{aligned} \tag{6}$$

Generalized Laurent series expansions of  $a$ ,  $b$ ,  $c$ ,  $d$  and  $e$  are

$$\begin{aligned} a &= \phi^{\alpha_1} \sum_{j=0}^{\infty} a_j(z, t) \phi^j \\ b &= \phi^{\alpha_2} \sum_{j=0}^{\infty} b_j(z, t) \phi^j \\ c &= \phi^{\alpha_3} \sum_{j=0}^{\infty} c_j(z, t) \phi^j \\ d &= \phi^{\alpha_4} \sum_{j=0}^{\infty} d_j(z, t) \phi^j \\ e &= \phi^{\alpha_5} \sum_{j=0}^{\infty} e_j(z, t) \phi^j \end{aligned} \tag{7}$$

with  $a_0, \dots, e_0 \neq 0$ , where  $\alpha_1, \dots, \alpha_5$  are negative integers and  $a_j, \dots, e_j$  are a set of expansion coefficients which are analytic in the neighbourhood of the non-characteristic singular manifold  $\phi(z, t) = t + \phi(z) = 0$ . Looking at the leading order,  $a \approx a_0 \phi^{\alpha_1}, \dots, e \approx e_0 \phi^{\alpha_5}$  are substituted in equation (6) and, upon balancing dominant terms, the following results are obtained:

$$\begin{aligned} \alpha_1 = \alpha_2 = -1 \quad \alpha_3 = \alpha_4 = \alpha_5 \\ a_0 b_0 = \frac{2\beta}{\gamma} \quad \text{and} \quad a_0 d_0 = b_0 c_0. \end{aligned} \tag{8}$$

Substituting full Laurent series and considering leading-order terms alone, we obtain the following equation:

$$\begin{pmatrix} A & -\gamma a_0^2 & 0 & 0 & 0 \\ -\gamma b_0^2 & A & 0 & 0 & 0 \\ f e_0 & 0 & j + \alpha_3 & 0 & f a_0 \\ 0 & f e_0 & 0 & j + \alpha_3 & f b_0 \\ 2f d_0 & 2f c_0 & 2f b_0 & 2f a_0 & -j - \alpha_3 \end{pmatrix} \begin{pmatrix} a_j \\ b_j \\ c_j \\ d_j \\ e_j \end{pmatrix} = 0 \quad (9)$$

where  $A = \beta(j-1)(j-2) - 2\gamma a_0 b_0$ .

On solving equation (9), the resonance values are found to be

$$j = -1, 0, 3, 4, -\alpha_3, -\alpha_3 \pm 2\sqrt{\frac{-2f^2\beta}{\gamma}}. \quad (10)$$

From careful analysis, we find that equation (10) admits a sufficient number of positive resonances only for the condition

$$-2f^2\beta = \gamma. \quad (11)$$

From the dominant terms it is clear that  $\alpha_3 = -2$ . The resonance value at  $j = -1$  represents the arbitrariness of the singularity manifold, while resonances at  $j = 0, 0$  are associated with the arbitrariness of the functions  $a_0, \dots, e_0$  (as seen in equation (8)). Upon substituting the full Laurent series of equation (7) in (6) and on collecting the coefficients of different powers of  $\phi$  we find that equation (6) admits a sufficient number of arbitrary functions at  $j = 2, 3, 4, 4$  and hence the system of equation (4) is expected to be integrable for the parametric condition (11).

### 3. Discussion and conclusion

If we perform the Painlevé analysis for the NLS equation (1), then the corresponding resonances will be

$$j = -1, 0, 3, 4 \quad (12)$$

so we can easily see that there is no parametric condition between  $\beta$  and  $\gamma$ . Hence GVD can take both positive and negative values, which corresponds to the dark and bright solitons in pure fibres described by the NLS equation alone.

However, from the Painlevé singularity structure analysis of the NLS–MB equations it is clear that the NLS–MB fibre system equation is integrable only for the parametric condition (11). The same condition for deriving the inverse scattering scheme for the NLS–MB equations has been reported in [1]. Here we have systematically derived that condition from the Painlevé analysis. From the parametric condition (11) it is clear that the parameters  $\beta$  and  $\gamma$  has to be of opposite sign. This means that the GVD parameter  $\beta$  has to be negative, which corresponds to the anomalous dispersion regime. It is a known fact that only a bright soliton is possible in anomalous dispersion [7]. Hence, from the parametric condition (11), it is clear that there can be only coexistence of bright solitons in an NLS–MB fibre system.

In an NLS fibre system both bright and dark solitons are possible. For bright and dark solitons one needs to have a sech and tanh type pulse intensity profile for propagation, but in two-level resonant media, the MB equation has only the sech type of soliton solution, which corresponds to the bright one. So the combined system of NLS and MB allows only the sech type of pulse profile in the erbium doped fibres.

From the phenomenon of SIT also, it is clear that the leading edge of the pulse intensity is utilized for the population inversion of the two-level system and this is used for the amplification of the trailing edge of the pulse to propagate as MB solitons. Only sech pulses will allow SIT

in the two-level media as it has the required profile of leading and trailing edges. This is the physical reason why only a bright soliton is possible in the coupled NLS–MB fibre system.

We like to stress the following important point regarding the possibility of optical solitons in the NLS–MB fibre system. From the Painlevé analysis it is clear that the system equation (4) is integrable only when the parametric condition (11) is satisfied. Hence there is no possibility of the propagation of both bright and dark optical solitons in the NLS–MB fibre system when the condition (11) is not satisfied. From the earlier discussions it is also obvious that there is no possibility of the propagation of dark optical solitons in the NLS–MB fibre system even when condition (11) is satisfied.

Thus in this article, we have proved the non-existence of dark solitons in the NLS–MB fibre system with the help of Painlevé analysis.

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